Fast Computation Method for Solving the Power Frequency Electric Field in Substation

ZHANG Zhan-Long, ZHU Zhen-Hai, XIAO Dong-Ping, ZHONG Jun
State Key Laboratory of Power Transmission Equipmentand System Security and New Technology, Chongqing University, Chongqing, 400030, China
zhangzl@cqu.edu.cn

Abstract —To solve the problems of power frequency electric field(PFEF) in substation, in this paper, a new approach to the fast multipole method in combination with higher order elements is presented. As the storage requirements and the computational costs are reduced from O(N^2) to approximately O(N), the large scale static problems with complex geometrical configuration can be solved on the computer with usual configuration.

I. INTRODUCTION

As many equipments such as insulating pillars, lighting arresters, switches and many other key equipments using frequency power are installed all together in the substation, the frequency power electric field(PFEF) environment in the substation is extremely adverse. It is an important topic to quantitatively analyze the PFEF problem. Boundary element method(BEM) is an increasingly popular approach to solve this sort of problems with high accuracy and fewer unknown quantities. But it will lead to a system of linear equations with a dense nonsymmetric matrix[**A**], and the storage and computational costs will increase significantly.

To reduce the order of the storage requirements and the computational costs, in this paper, a fast multipole boundary element method(FM-BEM) in combination with higher order elements is presented. With this method the matrix-vector-product will be obtained without the dense BEM-matrix, the storage requirements and the computational costs will be reduced from O(N^2) to approximately O(N)[1], hence the PFEF problems can be solved with high accuracy by using the computer with usual configuration.

II. THEORETICAL BACKGROUND AND FORMULATION

A. Formulation of the Problem and Solution with the higher order BEM

Suppose the electrostatic problems with conductors embedded in multiple, piecewise homogeneous dielectrics. The Dirichlet boundary conditions are the given potentials of the conductors and the Neumann boundary condition is the continuity of the normal of the dielectric displacement at the interfaces between two dielectrics. The boundary integral equation with the mixed boundary can be expressed,

$$C(x)\phi(x)+\int_{\partial\Omega}\frac{G(x,y)}{\partial n(y)}\phi(y)dS(y) = \int_{\partial\Omega}G(x,y)\frac{\partial\phi(y)}{\partial n(y)}dS(y) \qquad (1)$$

where $C(x)$=1 when the source point x in the boundary Ω; $C(x)$=1/2 when the source point x at the boundary Ω; $C(x)$ =0 when the source point x out of the boundary Ω; G(x,y) is the basis solution of 3D Laplacian.

Using eight nodded quadrilateral second order boundary elements to discrete all the boundary of computation domain to N elements[2]. We can obtain,

$$\frac{\phi(x_j)}{2}+\sum_{i=1}^{N}\int_{-1}^{1}\int_{-1}^{1}\frac{\partial G(x_j,y)}{\partial n}\sum_{l=1}^{k}N_l(\xi,\eta)\phi_l\left|J_i(\xi,\eta)\right|d\xi d\eta = \sum_{i=1}^{N}\int_{-1}^{1}\int_{-1}^{1}G(x_j,y)\sum_{l=1}^{k}N_l(\xi,\eta)q_l\left|J_i(\xi,\eta)\right|d\xi d\eta \qquad (2)$$

where $N_l(\xi,\eta)$ is the shape function, $\left|J_i(\xi,\eta)\right|$ is the Jacobian determinant, by rearranging the equation(2) can obtain a linear equations,

$$A_{N\times N}x_N=b_N \qquad (3)$$

where $A_{N\times N}$ is the coefficient matrix, x_N is the unknown vector, b_N is the known right-hand side vector.

B. Solution with the FM-BEM

Since solve the linear system of equations, we need to compute the system matrix[A]. It leads to a large number of computational costs. But the property of the matrix-vector-product allows us to compute the linear system of equations with the fast multipole method(FMM).

The FMM is based on series expansions of Green's function in spherical coordinates,

$$G(x,y)=\frac{1}{4\pi|x-y|}\cong\frac{1}{4\pi}\sum_{l=0}^{p}\sum_{m=-l}^{l}R_{lm}(y,y_0)\times S_{lm}(x,y_0) \qquad (4)$$

where $R_{lm}(y,y_0)$ and $S_{lm}(x,y_0)$ are the solid harmonics functions.

The implementation of the fast multipole method is using a smallest cube encloses all the elements, called root. Then subdivide this cube into eight equal sized cubes. If the number of elements in a cube is larger than a certain value, the cube, so called parent, is subdivided into eight smaller cubes, so called children, and then these cubes are subdivided and so on. If the number of elements in a cube is smaller than this certain value, the cube is only subdivided when all elements lie in one of its children[3]-[4].

In the first step the multipole coefficients of the parent can be obtained from a transformation of the multipole coefficients of its children.

$$M_{lm}(y')=\sum_{l'=0}^{l}\sum_{m'=-l'}^{l'}(-1)R_{l'm'}(y_0,y')M_{(l-l')(m-m')}(y_0) \tag{5}$$

In the next step the local coefficients of the cubes in the far-field are computed from the multipole coefficients of the cubes of the same level of subdivision, which have no common corner with the considered cube.

$$L_{lm}(x')\cong\sum_{l'=0}^{l}\sum_{m'=-n'}^{l'}(-1)^n\overline{S_{(l+l')(m+m')}(x',y')} \tag{6}$$
$$\times M_{l',m'}(y')$$

And then the local coefficients are transformed to local coefficients of the children.

$$L_{lm}(x_0)=\sum_{l'=0}^{\infty}\sum_{m'=-l'}^{l'}L_{l'm'}(x')R_{(l-l')(m-m')}(x_0,x') \tag{7}$$

Finally we can rewrite the equation(3).

$$A_{N\times N}x_N=\sum_{q}\sum_{i\in W_q}q_iE_{ji}+ \tag{8}$$
$$\sum_{q}\sum_{i\in W_q}\frac{1}{4\pi}\sum_{n=0}^{\infty}\sum_{m=-n}^{n}R_{n,m}(x,x_0)\times L_{n,m}(x_0)$$

While the elements that are close to the collocation point, the conventional direct BEM will be used, and the other elements can use the FMM to reduce the storage and computational costs.

III. NUMERICAL RESULTS

An example is chosen to show the accuracy of the FM-BEM in comparison with the BEM and the measurements. We consider the PFEF distribution produced by key devices of a 500kV outdoor substation, whose general view and essential dimensions are shown in Fig.1.

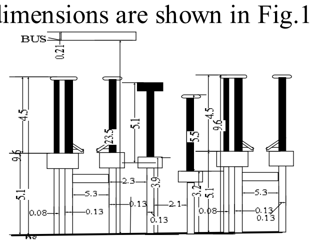

Fig.1. A part of key devices of switching field in 500 kV substation

The computational results are presented in Fig.2.

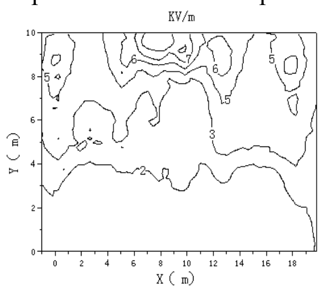

Fig.2. Potentiallines distribution in the vicinity of the key devices

The storage and calculation time versus element numbers of the FM-BEM and BEM are displayed in Fig.3 and Fig.4.

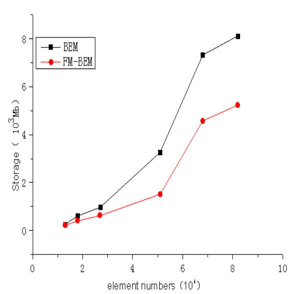

Fig.3. Storage in computer vs element numbers

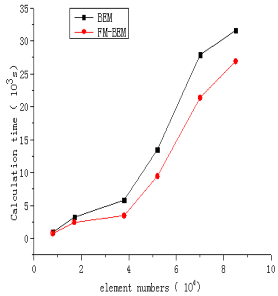

Fig.4. Calculation time vs element numbers

In order to verify the reliability of the algorithm, we measuremented the electric field in the 500kV substation where 1.8m far from the earth by electric meter. The comparison results are shown in Tab.1.

Tab.1 Comparison results of measurement and simulation by FM-BEM

x (m)	measurement (kv/m)	simulation (kv/m)	Relative error (%)
0.2	1.269	1.193	5.99
2	1.169	1.226	4.88
3.8	1.036	0.959	7.43
5.6	1.289	1.307	1.4
7.4	1.311	1.259	3.97
9.2	1.179	1.198	1.61
11	1.256	1.289	2.63
12.8	1.156	1.056	8.65
14.6	1.245	1.197	3.86
16.4	1.356	1.296	4.42
18.2	1.203	1.165	3.16
20	1.039	0.988	4.91

IV. CONCLUSION

The FM-BEM proposed in this paper is relatively efficient. In order to improve this method， some methods may embed in it, such as the generalized minimal residual algorithm(GMRES), which to solve general non-symmetric linear systems generated by FM[5] and the preconditioner to reduce the number of iteration steps.

V. REFERENCES

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